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Non-BPS Dp-brane in the background of NS5-branes on transverse $R^3 \times S^1$

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Abstract: This paper is devoted to the study of non-BPS Dp-branes in the presence of NS5-branes on the transverse $R^3 \times S^1$. We will formulate the tachyon effective action in this background and then we will discuss its properties. Then we will study the solutions of the equations of motion that describe lower dimensional BPS and non-BPS D-branes.

Keywords: D-branes, Tachyon Condensation.

Contents

1. Introduction

It is well known that type-IIA (IIB) string theories contain two types of D-branes: the BPS Dp-branes, which have even (odd)p in type-IIA (IIB) theories and unstable, non-BPS Dp-branes with odd (even)p in the type-IIA (IIB) case, for review see [1, 2]. Non-BPS D-branes are very important in string theory. For example, the BPS D-branes can be thought as solitons in the worldvolume theory of non-BPS ones [3, 4, 5]. However, as was stressed recently in [14] there are many open questions about them that remain unanswered. For example, it is very remarkable that many aspects of the tachyon dynamics can be captured by a spacetime effective action of Dirac-Born-Infeld (DBI) type [6]–[12]

$$S = -T_p^{\mathrm{non}} \int d^{p+1}\xi \frac{1}{\cosh \frac{T}{\sqrt{2}}} \sqrt{-\det G}\,, \tag{1.1}$$

where T_p^{non} is tension of non-BPS Dp-brane and G is induced metric on the brane

$$G_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu T \partial_\nu T + \partial_\mu Y^I \partial_\nu Y^I\,. \tag{1.2}$$

The scalar fields Y^I $(I = p+1, \dots, 9)$ living on the worldvolume of the brane parameterise its location in the transverse space. The form of the induced metric (1.2) suggests that the tachyon direction in field space should be treaded as an extra dimension of space, like Y^I, however then there is an important question: what is the meaning of the tachyon potential $V(T)$?

In two recent papers by D. Kutasov [13, 14][1] the precise analogy between the BPS D-brane moving in the background of NS5-branes and the tachyon dynamics on non-BPS Dp-brane was demonstrated. In particular, in [14] the dynamics of BPS D-brane propagating in the near horizon limit of $NS5$-branes with the transverse space $R^3 \times S^1$ was considered as an useful toy model of the non-BPS D-brane. It was shown that from the point of view

[1] Similar problems were discussed in [15]–[20].

of an observer living on the 5+1 dimensional worldvolume of fivebranes the BPS D-branes in the full theory give rise to two kinds of the objects in five dimensions. One consists D-branes whose worldvolume lies entirely inside the worldvolume of fivebranes. These D-branes are non-BPS-while both fivebranes and the BPS D-branes separately preserve 16 supersymmetries, a background that contains both of them breaks all supersymmetry. The second kind consists D-branes that wrap the circle transverse to the fivebranes. These D-branes preserve eight supercharges and are BPS. In summary, two kinds of D-branes emerge from different orientations of BPS D-branes in the space transverse to fivebranes.

In this paper we will continue the study of this interesting problem. Namely, we will consider a non-BPS D-brane embedded in the background of k $NS5$-branes on transverse $R^3 \times S^1$. We will explicitly show that the tachyon effective action in this background is invariant under special transformation that maps the tachyon mode T-that is presented on the worldvolume of a non-BPS Dp-brane even in the flat spacetime-to the new tachyon field $\mathcal{T}$-that arises from the field redefinition of the worldvolume field y that parameterises the position of a non-BPS Dp-brane on the circle S^1. The existence of this symmetry, even if its physical origin is unclear to us, really suggests that it is correct to consider the tachyon mode as an additional embedding coordinate. However the origin of this possible additional dimension is unclear at present.

After the discussion of the general properties of the tachyon effective action in the fivebranes background we will study some solutions of its equations of motion. We will show that these solutions describe both non-BPS and BPS lower dimensional D-branes that are embedded in the fivebrane background with the transverse space $R^3 \times S^1$. These solutions explicitly demonstrate that all branes in the fivebranes background arise through the tachyon condensation on the worldvolume of a non-BPS D-brane in the same way as D-branes in the flat spacetime can be thought as solitonic solutions on the higher dimensional non-BPS D-brane.

This paper is organised as follows. In the next section (2) we will study the properties of a non-BPS Dp-brane in the NS5-branes background with transverse space $R^3 \times S^1$. Then in section (3) we will analyse some solutions of the equation of motion and we will give their physical interpretation. Finally, in conclusion (4) we will outline our results and suggest possible extension of this work.

2. Tachyon effective action in the presence of NS5-branes on transverse $R^3 \times S^1$

To begin with we give a brief description of the system of k $NS5$-branes on transverse $R^3 \times S^1$ which we will label with coordinates $(\mathbf{Z}, Y)$ with $\mathbf{Z} = (Z^1, Z^2, Z^3) \in R^3$ and $Y \sim Y + 2\pi R$ where R is radius S^1. The fivebranes are located at points $\mathbf{Z} = Y = 0$. The background around them is [22]

$$\begin{aligned} ds^2 &= dx^\mu dx_\mu + H(\mathbf{Z}, Y)(d\mathbf{Z}^2 + dY^2)\,, \\ e^{2(\Phi-\Phi_0)} &= H(\mathbf{Z}, Y)\,, \end{aligned} \tag{2.1}$$

where $x^\mu \in R^{5,1}$ label the worldvolume of fivebranes and where Φ_0 is related to the string coupling constant g_s as $g_s = \exp \Phi_0$. The harmonic function H in (2.1) has the form

$$H = 1 + k \sum_{n=-\infty}^{\infty} \frac{1}{(Y - 2\pi Rn)^2 + \mathbf{Z}^2} \,. \tag{2.2}$$

We will be interested in the study of the system in the near-horizon limit that can be defined by rescaling all distances by the factor g_s

$$\mathbf{Z} = g_s \mathbf{z} \,, \qquad R = g_s r \,, \qquad Y = g_s y \tag{2.3}$$

and sending $g_s \to 0$ while keeping the rescaled distances $(\mathbf{z}, y, r)$ fixed. This leads to the background

$$ds^2 = dx^\mu dx_\mu + h(\mathbf{z}, y)(d\mathbf{z}^2 + dy^2) \,, \qquad e^{2\Phi} = h(\mathbf{z}, y) \tag{2.4}$$

with

$$h(\mathbf{z}, y) = k \sum_{n=-\infty}^{\infty} \frac{1}{(y - 2\pi rn)^2 + \mathbf{z}^2} = \frac{k}{2r|\mathbf{z}|} \frac{\sinh(\frac{|\mathbf{z}|}{r})}{\cosh(\frac{|\mathbf{z}|}{r}) - \cos(\frac{y}{r})} \,. \tag{2.5}$$

We now place non-BPS Dp-brane whose worldvolume is embedded entirely in $R^{5,1}$ in the geometry (2.4) and (2.5). The action for a non-BPS Dp-brane in this background takes the form

$$\begin{aligned} S &= -\int d^{p+1}\xi \frac{V(T)}{\sqrt{h(\mathbf{z}, y)}} \sqrt{-\det G_{\mu\nu}} \\ &= -\int d^{p+1}\xi \sqrt{-\det \eta} \frac{V(T)}{\sqrt{h(\mathbf{z}, y)}} \sqrt{\det(\mathbf{I} + \mathbf{M})} \,, \end{aligned} \tag{2.6}$$

where

$$G_{\mu\nu} = \eta_{\mu\nu} + h(\mathbf{z}, y) \left(\partial_\mu z^i \partial_\nu z^i + \partial_\mu y \partial_\nu y\right) + \partial_\mu T \partial_\nu T \tag{2.7}$$

and where we have also introduced $(n+1) \times (n+1)$ unit matrix $\mathbf{I}^\mu_\nu = \delta^\mu_\nu$ together with $(n+1) \times (n+1)$ matrix $\mathbf{M}$

$$\mathbf{M}^\mu_\nu = h(\mathbf{z}, y)(\partial^\mu z^i \partial_\nu z^i + \partial^\mu y \partial_\nu y) + \partial^\mu T \partial_\nu T \,. \tag{2.8}$$

The action (2.6) describes the non-BPS Dp-brane that is localised in the transverse space labelled with $\mathbf{z}, y$. As in the case of BPS Dp-brane studied in [14] we will be interested in the study of the dynamics of the mode y. For that reason we should show that $\mathbf{z}$ can be put in the values that solve their equations of motions that arise from (2.6)

$$\begin{aligned} &-\frac{1}{2h^{3/2}} \partial_{z^i} h V(T) \sqrt{\det(\mathbf{I} + \mathbf{M})} - \\ &-\partial_\kappa \left[\eta^{\kappa\mu} \sqrt{h} V(T) \partial_\nu z^i (\mathbf{I} + \mathbf{M})^{-1\nu}_\mu \sqrt{\det(\mathbf{I} + \mathbf{M})}\right] + \\ &+\frac{V(T)}{\sqrt{h}} \partial^\mu x^m \partial_\nu x^m \partial_{z^i} h (\mathbf{I} + \mathbf{M})^{-1\nu}_\mu \sqrt{\det(\mathbf{I} + \mathbf{M})} = 0 \,, \end{aligned} \tag{2.9}$$

where $x^m \equiv (\mathbf{z}, y)$. For constant $\mathbf{z}$ the equation of motion (2.9) takes simple form

$$\partial_{z^i} h = 0 \,. \tag{2.10}$$

Using the form of h given in (2.5) it is easy to see that this equation has the solution $z^i = 0$. Then one can place the fields z^i in their minimum at $z^i = 0$ and consider the dynamics of y and T only. Then the non-BPS Dp-brane action takes the form

$$S = -\int d^{p+1}\xi \frac{V(T)}{\sqrt{h(y)}}\sqrt{-\det(\eta_{\mu\nu} + h\partial_\mu y\partial_\nu y + \partial_\mu T\partial_\nu T)}\,, \tag{2.11}$$

where $V(T)$ is equal to

$$V(T) = \frac{T_p^{\mathrm{non}}}{\cosh\frac{T}{\sqrt{2}}}\,, \tag{2.12}$$

and where T_p^{non} is defined such that the tension of a non-BPS Dp-brane at flat spacetime is T_p^{non}/g_s. Note that T_p^{non} is related to the quantity that appears in the action for BPS Dp-brane as $T_p^{\mathrm{non}} = \sqrt{2}T_p^{\mathrm{BPS}}$.

Now, following [13, 14] we introduce "new" tachyon field $\mathcal{T}$ that is related to y through the relation

$$\frac{d\mathcal{T}}{dy} = \sqrt{h(y)} = \frac{\sqrt{k}}{2r\sin\frac{y}{2r}}\,. \tag{2.13}$$

This differential equation has the solution

$$e^{-\frac{\mathcal{T}}{\sqrt{k}}} = C_0\frac{\cos\frac{y}{4r}}{\sin\frac{y}{4r}}\,. \tag{2.14}$$

Now if we demand that for $y = \pi r$ the tachyon field $\mathcal{T}$ is equal to zero we get $C_0 = 1$. Then we obtain

$$h(y(\mathcal{T})) = \frac{1}{4r^2}\cosh^2\left(\frac{\mathcal{T}}{\sqrt{k}}\right) \tag{2.15}$$

and hence the tachyon effective action (2.11) can be written as

$$\begin{aligned}
S &= -\int d^{p+1}\xi \mathcal{V}(T,\mathcal{T})\sqrt{-\det(\eta_{\mu\nu} + \partial_\mu\mathcal{T}\partial_\nu\mathcal{T} + \partial_\mu T\partial_\nu T)} \\
&= -\int d^{p+1}\xi \mathcal{V}(T,\mathcal{T})\sqrt{\det(\mathbf{I}+\mathbf{M})}\,, \\
\mathcal{V}(\mathcal{T},T) &= \frac{\tau_p}{\cosh\frac{\mathcal{T}}{\sqrt{k}}\cosh\frac{T}{\sqrt{2}}}\,, \\
(\mathbf{I}+\mathbf{M})_\mu^\nu &= \delta_\mu^\nu + \eta^{\mu\kappa}\partial_\kappa\mathcal{T}\partial_\nu\mathcal{T} + \eta^{\mu\kappa}\partial_\kappa T\partial_\nu T\,,
\end{aligned} \tag{2.16}$$

where

$$\tau_p = \frac{2T_p^{\mathrm{non}}R}{\sqrt{k}g_s}\,. \tag{2.17}$$

Before we proceed to the solution of the equation of motions that arise from (2.16) we would like to say few words about symmetries of the action (2.16) that relate $\mathcal{T}$ with T. In particular, we see that this action is invariant under the transformation that maps T , $\mathcal{T}, A \equiv k$, $B \equiv 2$ into the new fields $T', \mathcal{T}'$ and new parameters A', B' given as

$$\mathcal{T}' = T\,, \qquad T' = \mathcal{T}\,, \qquad A' = B\,, \qquad B' = A \tag{2.18}$$

so that $S(T',\mathcal{T}',A',B')=S(T,\mathcal{T},A,B)$. In fact, one can find more general symmetry of the action (2.16). For that reason let us introduce two-component vectors X^I, Y^I , $I=1,2$ defined as

$$X=\left(\frac{1}{\sqrt{k}},\frac{1}{\sqrt{2}}\right), \qquad Y=\left(\frac{1}{\sqrt{k}},-\frac{1}{\sqrt{2}}\right) \tag{2.19}$$

that allow us to rewrite the tachyon potential $\mathcal{V}(T,\mathcal{T})$ as

$$\begin{aligned}\frac{\tau_p}{\cosh\frac{\mathcal{T}}{\sqrt{k}}\cosh\frac{T}{\sqrt{2}}}&=\frac{2\tau_p}{\left(\cosh(\frac{\mathcal{T}}{\sqrt{k}}+\frac{T}{\sqrt{2}})+\cosh(\frac{\mathcal{T}}{\sqrt{k}}-\frac{T}{\sqrt{2}})\right)}\\ &=\frac{2\tau_p}{(\cosh(X^I\delta_{IJ}\mathbf{T}^J)+\cosh(Y^I\delta_{IJ}\mathbf{T}^J))}\,,\end{aligned} \tag{2.20}$$

where we have introduced two-component vector $\mathbf{T}=(\mathcal{T},T)$. In this notation the tachyon effective action (2.11) takes the form

$$S=-\int d^{p+1}\xi\frac{2\tau_p}{(\cosh(X^I\delta_{IJ}\mathbf{T}^J)+\cosh(Y^I\delta_{IJ}\mathbf{T}^J))}\sqrt{-\det(\eta_{\mu\nu}+\partial_\mu\mathbf{T}^I\partial_\nu\mathbf{T}^J\delta_{IJ})}\,. \tag{2.21}$$

Now it is easy to see that this action is invariant under following transformations

$$X'^I=\Lambda^I_JX^J\,, \qquad Y'^I=\Lambda^I_JY^J\,, \qquad \mathbf{T}'^I=\Lambda^I_J\mathbf{T}^J\,, \tag{2.22}$$

where Λ^I_J obeys $\Lambda^I_K\delta_{IJ}\Lambda^J_L=\delta_{KL}$.

We mean that an existence of the transformation (2.22) is very attractive since it relates the tachyon field T with the field $\mathcal{T}$ and then in some sense suggests the geometrical nature of T. Unfortunately it is also clear that this form of symmetry is rather unusual and its possible physical origin is unclear at present. In fact we do not understand how T and $\mathcal{T}$ could be related in such a simple way when we know that their origin is completely different. Secondly, we also do not understand the physical meaning of the vectors X and Y defined above. On the other hand one can hope that discovery of all possible symmetries of the tachyon effective action could be helpful for better understanding of the meaning of the tachyon in string theory.

3. Solutions of the equations of motion

In this section we would like to study the equations of motion for T and $\mathcal{T}$ that arise from the action (2.16)

$$\begin{aligned}&-\frac{\sinh\frac{\mathcal{T}}{\sqrt{k}}}{\sqrt{k}\cosh^2\frac{\mathcal{T}}{\sqrt{k}}\cosh\frac{T}{\sqrt{2}}}\sqrt{\det(\mathbf{I}+\mathbf{M})}-\partial_\mu\left[\frac{1}{\cosh\frac{\mathcal{T}}{\sqrt{k}}\cosh\frac{T}{\sqrt{2}}}\frac{\eta^{\mu\kappa}\partial_\nu\mathcal{T}(\mathbf{I}+\mathbf{M})^{-1\nu}_{\kappa}}{\sqrt{\det(\mathbf{I}+\mathbf{M})}}\right]=0\,,\\ &-\frac{\sinh\frac{T}{\sqrt{2}}}{\sqrt{2}\cosh\frac{\mathcal{T}}{\sqrt{k}}\cosh^2\frac{T}{\sqrt{2}}}\sqrt{\det(\mathbf{I}+\mathbf{M})}-\partial_\mu\left[\frac{1}{\cosh\frac{\mathcal{T}}{\sqrt{k}}\cosh\frac{T}{\sqrt{2}}}\frac{\eta^{\mu\kappa}\partial_\nu T(\mathbf{I}+\mathbf{M})^{-1\nu}_{\kappa}}{\sqrt{\det(\mathbf{I}+\mathbf{M})}}\right]=0\,.\end{aligned} \tag{3.1}$$

For next purposes it will be also useful to calculate the stress energy tensor from the action (2.16). In order to do this we replace the flat worldvolume metric $\eta_{\mu\nu}$ with arbitrary metric $g_{\mu\nu}$. Using now the fact that for the action of the form

$$S = -\int d^{p+1}\xi\sqrt{-g}\mathcal{L} \tag{3.2}$$

the stress energy tensor $T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}$ is equal to

$$T_{\mu\nu} = -g_{\mu\nu}\mathcal{L} + 2\frac{\delta\mathcal{L}}{\delta g^{\mu\nu}} \tag{3.3}$$

we obtain from (2.16)

$$T_{\mu\nu} = -\eta_{\mu\nu}\mathcal{V}\sqrt{\det(\mathbf{I}+\mathbf{M})} + \mathcal{V}(\partial_\nu\mathcal{T}\partial_\kappa\mathcal{T} + \partial_\nu T\partial_\kappa T)(\mathbf{I}+\mathbf{M})^{-1\kappa}_{\mu}\sqrt{\det(\mathbf{I}+\mathbf{M})}\,. \tag{3.4}$$

Now we are ready to study some solutions of the equation of motions (3.1). We begin with the case when $\mathcal{T}$ is time dependent while the tachyon T depends on one spatial coordinate $x \equiv \xi^1$. Then

$$\mathbf{M} = \begin{pmatrix} -\dot{\mathcal{T}}^2 & 0 & 0 \\ 0 & T'^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \det(\mathbf{I}+\mathbf{M}) = (1-\dot{\mathcal{T}}^2)(1+T'^2)\,, \qquad T' \equiv \frac{dT}{dx} \tag{3.5}$$

and hence components of the stress energy tensor are equal to

$$\begin{aligned}
T_{00} &= \frac{\mathcal{V}(1+T'^2)}{\sqrt{(1-\dot{\mathcal{T}}^2)(1+T'^2)}}\,, \qquad T_{0i} = 0\,, \qquad i = 1,\dots,p \\
T_{xx} &= -\frac{\mathcal{V}(1-\dot{\mathcal{T}}^2)}{\sqrt{(1-\dot{\mathcal{T}}^2)(1+T'^2)}}\,, \qquad T_{ix} = T_{xi} = 0\,, \qquad i = 2,\dots,p \\
T_{ij} &= -\delta_{ij}\mathcal{V}\sqrt{(1-\dot{\mathcal{T}}^2)(1+T'^2)}\,, \qquad i,j = 2,\dots,p\,.
\end{aligned} \tag{3.6}$$

Now for the matrix $\mathbf{M}$ given in (3.5) the equations of motion (3.1) take the form

$$\begin{aligned}
&\frac{\sqrt{1+T'^2}}{\cosh\frac{T}{\sqrt{2}}}\left[-\frac{\sinh\frac{\mathcal{T}}{\sqrt{k}}\sqrt{1-\dot{\mathcal{T}}^2}}{\sqrt{k}\cosh^2\frac{\mathcal{T}}{\sqrt{k}}} + \partial_0\left(\frac{1}{\cosh\frac{\mathcal{T}}{\sqrt{k}}}\frac{\partial_0\mathcal{T}}{\sqrt{1-\dot{\mathcal{T}}^2}}\right)\right] = 0 \\
&\frac{\sqrt{1-\dot{\mathcal{T}}^2}}{\cosh\frac{\mathcal{T}}{\sqrt{k}}}\left[-\frac{\sinh\frac{T}{\sqrt{2}}\sqrt{1+T'^2}}{\sqrt{2}\cosh^2\frac{T}{\sqrt{2}}} - \partial_x\left(\frac{1}{\cosh\frac{T}{\sqrt{2}}}\frac{\partial_x T}{\sqrt{1+T'^2}}\right)\right] = 0\,.
\end{aligned} \tag{3.7}$$

These expressions explicitly show that $\mathcal{T}$ and T decouple. In particular, let us consider the last equation in (3.7). If we define

$$V(T,c) = \frac{1}{\cosh\frac{T}{\sqrt{c}}} \tag{3.8}$$

then the expression in the bracket can be written as

$$\frac{\delta V(T,2)}{\delta T}\sqrt{1+T'^2}-\partial_x\left(\frac{V(T,2)\partial_x T}{\sqrt{1+T'^2}}\right)=0\Rightarrow$$
$$\Rightarrow\partial_x\left(\frac{V(T,2)}{\sqrt{1+T'^2}}\right)=0\Rightarrow\frac{V(T,2)}{\sqrt{1+T'^2}}=p\,, \tag{3.9}$$

where p is an integration constant. Using now the specific form of the potential $V(T,2)$ given in (3.8) it is easy to find the dependence of T on x

$$\sinh\frac{T}{\sqrt{2}}=\frac{\sqrt{1-p^2}}{p}\sin\frac{x}{\sqrt{2}}\,. \tag{3.10}$$

For such a configuration the spatial dependent energy density is equal to

$$\begin{aligned}\rho(x)=T_{00}(x)&=\frac{\tau_p V(\mathcal{T},k)}{\sqrt{1-\dot{\mathcal{T}}^2}}V(T,2)\sqrt{1+T'^2}\\&=\frac{\tau_p V(\mathcal{T},k)}{\sqrt{1-\dot{\mathcal{T}}^2}}\frac{p}{p^2+(1-p)^2\sin^2\frac{x}{\sqrt{2}}}\,.\end{aligned} \tag{3.11}$$

The physical interpretation of this solution is in terms of an array of D$(p-1)$-branes and D$(p-1)$-antibranes [23, 24, 25] that move toward to the worldvolume of $NS5$-branes. To see this more explicitly let us now solve the equation of motion for $\mathcal{T}$ (3.7) that can be written as

$$-\frac{\delta V(\mathcal{T},k)}{\delta\mathcal{T}}\sqrt{1-\dot{\mathcal{T}}^2}+\partial_0\left(\frac{V(\mathcal{T},k)\dot{\mathcal{T}}}{\sqrt{1-\dot{\mathcal{T}}^2}}\right)=0\Rightarrow$$
$$\Rightarrow\partial_0\left(\frac{V(\mathcal{T},k)}{\sqrt{1-\dot{\mathcal{T}}^2}}\right)=0\Rightarrow\frac{V(\mathcal{T},k)}{\sqrt{1-\dot{\mathcal{T}}^2}}=e\,. \tag{3.12}$$

Using again (3.8) we obtain

$$\sinh\frac{\mathcal{T}}{\sqrt{k}}=\frac{\sqrt{e^2-1}}{e}\sinh\left(\frac{t}{\sqrt{k}}+t_0\right)\,. \tag{3.13}$$

We can fix the constant t_0 from the requirement that at time $t=0$ the non-BPS Dp-brane sits at the point $y=\pi r$ $(\mathcal{T}=0)$. As a result t_0 should be equal to zero. Then

$$e=\frac{1}{\sqrt{1-\dot{\mathcal{T}}_0^2}}\Rightarrow\dot{\mathcal{T}}_0^2=\frac{\sqrt{e^2-1}}{e^2} \tag{3.14}$$

and hence e is related to the velocity $\dot{\mathcal{T}}$ at time $t=0$.

In summary, we have got the solution where the spatial dependent tachyon condensation T results to an emergence of the array of D$(p-1)$-branes and D$(p-1)$-antibranes. Since non-BPS Dp-branes in type-IIA (IIB)theories have odd(even) p the tachyon condensation leads to the emergence of BPS D$(p-1)$-branes with even (odd) p. However the configuration when these D-branes are inserted in the background of $NS5$-branes is unstable and hence these D-branes are moving towards to the worldvolume of NS5-branes. This situation is described by time dependent condensation of field $\mathcal{T}$.

Let us now consider the situation when $\mathcal{T}$ is function of x and T is function of t. It is clear that we could proceed in the same way as in the previous example however in order to obtain clear physical meaning of the resulting configuration it will be useful to construct the singular kink following the analysis performed in [21]. First of all, the equations of motion (3.1) for $\mathcal{T} = \mathcal{T}(x)$ and for $T = T(t)$ take the form

$$\begin{aligned}
&\frac{\sqrt{1-\dot{T}^2}}{\cosh\frac{T}{\sqrt{2}}}\left[-\frac{\sinh\frac{\mathcal{T}}{\sqrt{k}}\sqrt{1+\mathcal{T}'^2}}{\sqrt{k}\cosh^2\frac{\mathcal{T}}{\sqrt{k}}}-\partial_x\left(\frac{1}{\cosh\frac{\mathcal{T}}{\sqrt{k}}}\frac{\partial_x\mathcal{T}}{\sqrt{1+\mathcal{T}'^2}}\right)\right]=0\,,\\
&\frac{\sqrt{1+\mathcal{T}'^2}}{\cosh\frac{\mathcal{T}}{\sqrt{k}}}\left[-\frac{\sinh\frac{T}{\sqrt{2}}\sqrt{1-\dot{T}^2}}{\sqrt{2}\cosh^2\frac{T}{\sqrt{2}}}+\partial_0\left(\frac{1}{\cosh\frac{T}{\sqrt{2}}}\frac{\partial_0 T}{\sqrt{1-\dot{T}^2}}\right)\right]=0\,.
\end{aligned}\tag{3.15}$$

Now the equation of motion for $\mathcal{T}$ implies

$$\partial_x\left(\frac{V(\mathcal{T},k)}{\sqrt{1+\mathcal{T}'^2}}\right)=0\tag{3.16}$$

that means that the expression in the bracket does not depend on x. Since for a kink solution $\mathcal{T}\to\pm\infty$ as $x\to\pm\infty$ and $V(\mathcal{T},k)\to 0$ in this limit we obtain that the expression in the bracket vanishes for $x\to\infty$ and from its independence on x it implies that it vanishes everywhere. This in turn implies that we should have

$$\mathcal{T}=\pm\infty \qquad \text{or} \qquad \partial_x\mathcal{T}=\infty \qquad \text{(or both)} \qquad \text{for all } x\,.\tag{3.17}$$

Clearly this solution looks singular. We will show, following [21], that this solution has finite energy density that is localised on codimension one subspace however the interpretation is slightly different than in the case of the tachyon kink on non-BPS Dp-brane in flat spacetime.

To see this let us consider the field configuration

$$\mathcal{T}(x)=f(ax)\,,\qquad f(u)=-f(-u)\,,\qquad f'(u)>0\forall x\,,\qquad f(\pm\infty)=\pm\infty\tag{3.18}$$

that in the limit $a\to\infty$ looks singular as expected. For this solution however we get

$$\frac{V(\mathcal{T},k)}{\sqrt{1+\mathcal{T}'^2}}=\frac{V(f(ax),k)}{\sqrt{1+a^2f'^2(ax)}}\tag{3.19}$$

that vanishes everywhere at the limit $a\to\infty$ since the numerator vanishes (except at $x=0$) and the denominator blows up everywhere. Using this solution it is easy to find other components of the stress energy tensor

$$\begin{aligned}
T_{00}(x)&=\frac{\tau_pV(T,2)}{\sqrt{1-\dot{T}^2}}V(\mathcal{T},k)\sqrt{1+\mathcal{T}'^2}\\
&=\frac{\tau_pV(T,2)}{\sqrt{1-\dot{T}^2}}V(f(ax),k)af'(ax)\,,\\
T_{ij}(x)&=-\delta_{ij}\tau_pV(T,2)\sqrt{1-\dot{T}^2}V(\mathcal{T},k)\sqrt{1+\mathcal{T}'^2}\\
&=-\delta_{ij}\tau_pV(T,2)\sqrt{1-\dot{T}^2}V(f(ax),k)af'(ax)
\end{aligned}\tag{3.20}$$

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in the limit $a \to \infty$. Then the integrated T_{00} , T_{ij} associated with the codimension one solution are equal to

$$T_{00}^{\mathrm{kink}} = \int dx T_{00} = \frac{\tau_p V(T,2)}{\sqrt{1-\dot{T}^2}} \int dx V(f(ax),k) a f'(ax) = \frac{\tau_p V(T,2)}{\sqrt{1-\dot{T}^2}} \int dy V(y,k) dy \,,$$
$$T_{ij}^{\mathrm{kink}} = -\delta_{ij} \tau_p V(T,2) \sqrt{1-\dot{T}^2} \int V(y) dy \,, \tag{3.21}$$

where $y = f(ax)$. Thus $T_{\alpha\beta}^{\mathrm{kink}}$, $\alpha, \beta = 0, 2, \dots, p$ depend on V and not on the form of $f(u)$. It is clear from the exponential fall off the function of V that most of the contribution is contained in the finite range of y. In fact, in the limit $a \to \infty$ the stress energy tensor $T_{\alpha\beta}^{\mathrm{kink}}$ is localised on codimension one D$(p-1)$-brane with the tension given as

$$T_{p-1} = \tau_p \int dy V(y,k) = \tau_p \int \frac{dy}{\cosh \frac{y}{\sqrt{k}}} = \frac{(2\pi)\sqrt{k}}{2} \tau_p = \frac{2\pi T_p^{\mathrm{non}} R}{g_s} \tag{3.22}$$

using the fact that τ_p is equal to

$$\tau_p = \frac{2 T_p^{\mathrm{non}} R}{\sqrt{k} g_s} \,. \tag{3.23}$$

Finally we obtain

$$T_{00}^{\mathrm{kink}} = \delta(x) T_{p-1} \frac{V(T,2)}{\sqrt{1-\dot{T}^2}} \,, \qquad T_{ij} = -\delta(x) \delta_{ij} T_{p-1} V(T,2) \sqrt{1-\dot{T}^2} \,. \tag{3.24}$$

The geometrical meaning of this solution is clear [14]. Since $\mathcal{T}$ is directly related to the coordinate y that parameterises the position of a non-BPS Dp-brane on the transverse circle, the singular kink solution corresponds to non-BPS D-brane that sits on top of the fivebranes for all $x < 0$ then at $x = 0$ goes around the y circle and then back to the fivebranes at $y = 2\pi R$ where it stays for all $x > 0$. This describes non-BPS Dp-brane wrapped around the transverse circle. Then the time dependent solution of the equation of motion for T

$$\sinh \frac{T}{\sqrt{2}} = \sinh \frac{t}{\sqrt{2}} \tag{3.25}$$

describes the annihilation of this Dp-brane to the closed string vacuum [1].

To obtain BPS-like D-brane that is stable in the NS5-brane background we should consider the situation when both $\mathcal{T}$ and T are spatial dependent. For that reason we take following ansatz

$$T(x^1) = f_1(x^1) \,, \qquad x^1 \equiv \xi^{p-1} \,, \qquad \mathcal{T}(x^2) = f_2(x^2) \,, \qquad x^2 \equiv \xi^p \,, \tag{3.26}$$

where $f_i(u)$ are functions with the properties given in (3.18). For this ansatz the matrix $\mathbf{I} + \mathbf{M}$ takes the form

$$\mathbf{I} + \mathbf{M} = \begin{pmatrix} \mathbf{I}_{(p-1)\times(p-1)} & 0 & 0 \\ 0 & 1 + (\partial_1 T)^2 & 0 \\ 0 & 0 & 1 + (\partial_2 \mathcal{T})^2 \end{pmatrix} , \tag{3.27}$$

where $\partial_1 \equiv \partial_{x^1}$, $\partial_2 \equiv \partial_{x^2}$. Then the components of the stress energy tensor are equal to

$$\begin{aligned} T_{00} &= \tau_p V(T,2)V(\mathcal{T},k)\sqrt{(1+(\partial_2\mathcal{T})^2)(1+(\partial_1 T)^2)}\,, \qquad T_{0i}=0\,, \qquad i=1,\dots,p\,, \\ T_{x^1x^1} &= -\tau_p V(\mathcal{T},k)\sqrt{1+(\partial_2\mathcal{T})^2}\frac{V(T,2)}{\sqrt{1+(\partial_1 T)^2}}\,, \qquad T_{x^1 i}=0\,, \qquad i=1,\dots,p-1 \\ T_{x^2x^2} &= \tau_p V(T,2)\sqrt{1+(\partial_1 T)^2}\frac{V(\mathcal{T},k)}{\sqrt{1+(\partial_2\mathcal{T})^2}}\,, \qquad T_{x^2 i}=0\,, \qquad i=1,\dots,p-2\,, \\ T_{ij} &= -\delta_{ij}\tau_p V(T,2)V(\mathcal{T},k)\sqrt{(1+(\partial_2\mathcal{T})^2)(1+(\partial_1 T)^2)}\,, \qquad i,j=1,2,\dots,p-2\,. \end{aligned} \tag{3.28}$$

Now the conservation of the stress energy tensor implies

$$\partial_{x^1}T_{x^1x^1}=0\,, \qquad \partial_{x^2}T_{x^2x^2}=0\,. \tag{3.29}$$

In other words, $T_{x^1x^1}$ does not depend on x^1 and $T_{x^2x^2}$ does not depend on x^2. Since for $x^1 \to \infty$ $V(T,2) \to 0$ and using the same arguments as in the case given above we get that $T_{x^1x^1}$ is equal to zero for all x^1. In the same way one can argue that $T_{x^2x^2}=0$ for all x^2. Then the ansatz (3.26) has following physical interpretation: The condensation of the tachyon field T leads to the emergence of BPS D$(p-1)$-brane localised at the point $x^1=0$ on the worldvolume of non-BPS Dp-brane. Then the next condensation of the field $\mathcal{T}$ describes D$(p-1)$-brane that for $x^2<0$ sits at the worldvolume of NS5-branes at $y=0$ at the point $x^2=0$ wraps the transverse circle back to $y=2\pi R$ and then it sits on the worldvolume of NS5-branes for $x^2>0$. In other words, this tachyon condensation leads to the BPS D$(p-1)$-brane that wraps the transverse circle. As is well known such a configuration is stable as opposite to the case of the BPS D-brane whose worldvolume is parallel with the worldvolume of the fivebranes.

In order to further support this picture we will calculate the stress energy tensor corresponding to this D$(p-1)$-brane

$$\begin{aligned} T^{D(p-1)}_{\alpha\beta} &= \int dx^1 dx^2 T_{\alpha\beta} \\ &= -\eta_{\alpha\beta}\tau_p\int dx^1 V(T,2)\sqrt{1+T'^2}\int dx^2 V(\mathcal{T},k)\sqrt{1+\mathcal{T}'^2} \\ &= -\eta_{\alpha\beta}\tau_p\int V(y^1,2)dy^1\int V(y^2,k)dy^2\,, \qquad y^i=f_i(a_i x^i)\,, \qquad i=1,2\,, \end{aligned} \tag{3.30}$$

where α, $\beta=0,1,\dots,p-2$. Since in the limit $a_i \to \infty$ the tachyon potential vanishes almost everywhere except at the point $x^i=0$ we can write the resulting stress energy tensor as

$$T_{\alpha\beta} = -\eta_{\alpha\beta}\delta(x^1)\delta(x^2)T_{p-1} \tag{3.31}$$

where

$$T_{p-1}=\tau_p\int dy^1\frac{1}{\cosh\frac{y^1}{\sqrt{2}}}\int dy^2\frac{1}{\cosh\frac{y^2}{\sqrt{k}}} = \frac{T^{\mathrm{BPS}}_{p-1}}{g_s}2\pi R \tag{3.32}$$

that is exactly an energy of BPS D$(p-1)$-brane wrapped around the circle with radius R.

Now we would like to determine the effective action for translation zero modes of this solution following the analysis performed in [21]. For that reason we will consider the ansatz for T and $\mathcal{T}$

$$T(x^1,\xi) = f_1(a_1(x^1 - t^1(\xi)) \,, \qquad \mathcal{T}(x^2,\xi) = f_2(a_2(x^2 - t^2(\xi)) \,, \tag{3.33}$$

where we have denoted ξ^α, $\alpha = 0, 1, \ldots, p-2$ the coordinates tangential to the kink worldvolume. For such a configuration we get

$$\begin{aligned}
\mathbf{A}_{x^1x^1} &= 1 + a_1^2 f_1'^2 \,, \qquad \mathbf{A}_{x^2x^2} = 1 + a_2^2 f_2'^2 \,, \\
\mathbf{A}_{\alpha x^1} &= \mathbf{A}_{x^1\alpha} = -a_1^2 f_1'^2 \partial_\alpha t^1 \,, \qquad \mathbf{A}_{\alpha x^2} = \mathbf{A}_{x^2\alpha} = -a_2^2 f_2'^2 \partial_\alpha t^2 \,, \\
\mathbf{A}_{\alpha\beta} &= (a_1^2 f_1'^2 - 1)\partial_\alpha t^1 \partial_\beta t^1 + (a_2^2 f_2'^2 - 1)\partial_\alpha t^2 \partial_\beta t^2 + \mathbf{a}_{\alpha\beta} \,, \\
\mathbf{a}_{\alpha\beta} &= \eta_{\alpha\beta} + \partial_\alpha t^i \partial_\beta t^i \,.
\end{aligned} \tag{3.34}$$

Let us now define following matrices

$$\begin{aligned}
\hat{\mathbf{A}}_{\mu\beta} &= \mathbf{A}_{\mu\beta} + \mathbf{A}_{\mu x^1}\partial_\beta t^1 + \mathbf{A}_{\mu x^2}\partial_\beta t^2 \,, \qquad \hat{\mathbf{A}}_{\mu x^1} = \mathbf{A}_{\mu x^1} \,, \qquad \hat{\mathbf{A}}_{\mu x^2} = \mathbf{A}_{\mu x^2} \,, \\
\tilde{\mathbf{A}}_{\alpha\nu} &= \hat{\mathbf{A}}_{\alpha\nu} + \hat{\mathbf{A}}_{x^1\nu}\partial_\alpha t^1 + \hat{\mathbf{A}}_{x^2\nu}\partial_\alpha t^2 \,, \qquad \tilde{\mathbf{A}}_{x^1\nu} = \hat{\mathbf{A}}_{x^1\nu} \,, \qquad \tilde{\mathbf{A}}_{x^2\nu} = \hat{\mathbf{A}}_{x^2\nu}
\end{aligned} \tag{3.35}$$

that obey

$$\det \hat{\mathbf{A}} = \det \tilde{\mathbf{A}} = \det \mathbf{A} \,. \tag{3.36}$$

On the other hand the explicit calculation gives

$$\begin{aligned}
\tilde{\mathbf{A}}_{\alpha\beta} &= \mathbf{a}_{\alpha\beta} \,, \qquad \tilde{\mathbf{A}}_{x^1\alpha} = \tilde{\mathbf{A}}_{\alpha x^1} = \partial_\alpha t^1 \,, \qquad \tilde{\mathbf{A}}_{x^2\alpha} = \tilde{\mathbf{A}}_{\alpha x^2} = \partial_\alpha t^2 \,, \\
\tilde{\mathbf{A}}_{x^1x^1} &= 1 + a_1^2 f_1'^2 \,, \qquad \tilde{\mathbf{A}}_{x^2x^2} = 1 + a_2^2 f_2'^2
\end{aligned} \tag{3.37}$$

and hence the determinant $\det \tilde{\mathbf{A}}$ for large a_1, a_2 takes the form

$$\det \tilde{\mathbf{A}} = a_1^2 f_1'^2 a_2^2 f_2'^2 \left[\det \mathbf{a}_{\alpha\beta} + \mathcal{O}\left(\frac{1}{a_1^2}\right) + \mathcal{O}\left(\frac{1}{a_2^2}\right)\right] . \tag{3.38}$$

Substituting this expression into the effective action we get

$$\begin{aligned}
S &= -\tau_p \int d^{p-1}\xi \sqrt{-\det \mathbf{a}} \int dx^1 dx^2 V(f_1(a_1), 2) V(f_2(a_2, k)) a_1 f_1'(a_1 x^1) a_2 f_2'(a_2 x^2) \\
&= -\frac{T_{p-1}^{\mathrm{BPS}} 2\pi R}{g_s} \int d^{p-1}\xi \sqrt{-\det \mathbf{a}}
\end{aligned} \tag{3.39}$$

that is the right form of the action for zero modes describing transverse fluctuations of BPS D$(p-1)$-brane that wraps the circle with radius R.

4. Conclusion

In this paper we have studied the dynamics of a non-BPS Dp-brane in the background of k $NS5$-branes on transverse $R^3 \times S^1$, following paper [14], where the dynamics of a BPS Dp-brane in this background was discussed. The main motivation was to clarify

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the relation between the true tachyon mode on the worldvolume of a non-BPS Dp-brane-that expresses an instability of this object even in flat spacetime background-and the new tachyon field that arises from the redefinition of the mode that describes the position of the non-BPS D-brane on a transverse circle S^1. We have found that a non-BPS Dp-brane in this background is invariant under the exchange T with $\mathcal{T}$ on condition that the numerical factors in the tachyon potentials are exchanged as well. This fact is clearly very puzzling since while k is the number of $NS5$-branes and hence has clear physical meaning the factor 2 in the tachyon potential V does not have such a clear physical interpretation. In the same way it is not clear whether the extended symmetry between $T, \mathcal{T}$ and parameters in the tachyon potentials that was discussed in section (2) has some physical meaning or whether this is only pure coincidence. On the other hand we mean that the idea that the tachyon on the worldvolume of a non-BPS Dp-brane could have geometrical origin is very intriguing and certainly deserve to be investigated further.

Then in section (3) we have studied the solutions of the tachyon effective action in the background defined above. For the spatial dependent tachyon T and time dependent tachyon $\mathcal{T}$ this solution describes collection of D$(p-1)$-branes and D$(p-1)$-antibranes that move towards to the worldvolume of fivebranes. On the other hand for the time dependent T and spatial dependent $\mathcal{T}$ we have considered the solution that corresponds to the emergence of a non-BPS D$(p-1)$-brane that wraps the transverse circle and that further annihilates in the process of the time dependent tachyon condensation. An finally, we have constructed solutions where both T and $\mathcal{T}$ were spatial dependent. We have then argued that this configuration describes BPS D$(p-1)$-brane wrapped around transverse circle S^1.

We hope that this modest contribution to the study of the dynamics of BPS and non-BPS D-branes in the NS5-branes background that we have performed in this paper could be helpful for further research of this very interesting subject and for better understanding of the role of the tachyon in string theory.

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